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The following calculation progression chart is to be used in conjunction with the Maths and Reasoning Policy and the chart of progression in use of the Bar Method. The examples below are for guidance for class teachers and not exclusive or exhaustive. Along with the charts below the NCETM documents ‘Mastery Assessment’, published for each year group, can be used to give more detail on images to use, how to progress children’s use of jottings and how to introduce abstract concepts alongside pictorial representation. This document is a working document and can be printed and annotated by teachers to support planning and teaching as necessary, and teachers may use elements from the previous and next year’s suggested imagery to support less secure pupils and pupils working at greater depth.

With regards to progression in use of the Bar Method images here are for guidance of class teachers and can be represented electronically, drawn by hand, drawn with whiteboard pens or in children’s books by the teacher. Children should be taught that the Bar Method is a way of drawing a problem and not a calculation method. That is why it is separated here from the calculation policy.

#### IHS Calculation Progression Policy

#### PART 1: ADDITION R-yr6

#### STATUTORY EXPECTATIONS (NC 2014) Rapid Recall/Mental Calculations Non-statutory guidance

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| YR | Count … from 1-20 … and say which no. is 1 more than a given no. Using quantities objects, + two U nos and count on to find the answer. [Expected] Estimate no. of objects; check quantities by counting up to 20.[Exceeding] | Practical (sometimes recorded using ICT or by adults).  Hannah … listed how many girls and how many boys were outside. [She] was able to say that “There are 5 girls and 4 boys. That’s 9 altogether”.  When playing in the shop Christopher used his shopping list to add 2 amounts. He said “the beans are 5 pence and the bananas are 3 pence, altogether that is 8 pence.”  [[EYFS Profile exemplifications, STA](http://media.education.gov.uk/assets/files/pdf/e/elg11%20numbers.pdf)] | | | | | | [http://tse1.mm.bing.net/th?&id=OIP.Mafcfa9567fc45361a908ad2f06e29ddco0&w=300&h=225&c=0&pid=1.9&rs=0&p=0&r=0](http://www.bing.com/images/search?q=counting+on+a+bead+string&view=detailv2&&id=3BBD96BF4EA4474AB45D38DF97750A00E69A1FD7&selectedIndex=7&ccid=r8%2bpVn/E&simid=608019735211607445&thid=OIP.Mafcfa9567fc45361a908ad2f06e29ddco0)  Pictures/Objects  Bead strings  I eat 2 cakes and my friend eats 3.  How many cakes did we eat altogether?  **ADULTS MODEL ALL RECORDING**  For example, the above problem might be recorded as:  *2 + 3 = 5* | | | | | | | | | | | Symbolic  8 people are on the bus. 5 more get on at the next stop.  How many people are on the bus now  [Might be recorded as: *8 + 5 = 13*] | | | | | Begin to learn addition facts for each number to 5, extend to 10 if ready. | *Children use concrete objects, including Numicon pegs to count and add.* |
| Y1 | Add one-digit and two-digit numbers to 20 (9 + 9), including zero  Read/write/interpret statements involving addition (+) and equals (=) signs. | Pupils use concrete objects and pictorial representations  (e.g.: Numicon, Base 10, Unifix)  *Problems should include terms: put together, add, altogether, total, take away, distance between, more than and less than, so pupils develop concept of +/- and use operations flexibly.* | | | Use cubes/ counters and numberlines on the Interactive Board.  Pictures/  Symbolic  Numicon and Numicon tens number line: | | | | | | Visual  (modelled using bead strings/Numicon number line)  13 + 5 = 18    Begin by counting on in 1s to add: | | | | | | Progress to bigger, efficient jumps and use Numicon and number line (Concrete)  5+4 = 9  [Image result for basic adding on a numberline](https://www.google.co.uk/imgres?imgurl=https://lh6.googleusercontent.com/Mkv6flS2s37bvMCObHor7ZEJPbV8NIuv7AOROGTzpXrbQfw9w5wpBkxcTUVENmZwClrr78ue0Pop_3ZxO7C9OwZTErOvnWRYy0xtyYP6iu-4-GBKPbQ&imgrefurl=http://study-video.com/en/free-mathematics-courses/21-arithmetic-course/1366-addition-using-the-number-line&docid=mPXqBazbR86Y5M&tbnid=K0ykeV5PQjiSQM:&vet=1&w=924&h=420&safe=strict&bih=463&biw=784&q=basic%20adding%20on%20a%20numberline&ved=0ahUKEwjgx8fLnsnRAhWnI8AKHcSqBhUQMwgrKA8wDw&iact=mrc&uact=8)  [jumps may be in 1s at first]  Bridging ten- apply number bond to ten knowledge:  8+7= | | | | | | Use known facts/partitioning  8 + 5 + 13  8 + 2 = 10  10 + 3 = 13  Partition practically:  [Image result for adding 2 digit numbers using Numicon](https://www.google.co.uk/imgres?imgurl=https://s-media-cache-ak0.pinimg.com/564x/e0/ba/95/e0ba95bf409f18ae8a81b89e55653d54.jpg&imgrefurl=https://uk.pinterest.com/pin/458382068295658216/&docid=RhiXNzNhxIQnyM&tbnid=dVC9cEYrZr7ptM:&vet=1&w=463&h=595&safe=strict&bih=463&biw=784&q=adding%202%20digit%20numbers%20using%20Numicon&ved=0ahUKEwj439yxoMnRAhUlBsAKHfeWBxkQMwg-KBswGw&iact=mrc&uact=8) | Represent/use number bonds (and related subtraction facts) within 20.  Missing number problems  (eg 16 = ? + 9)  [Image result for adding using numicon](https://www.google.co.uk/imgres?imgurl=https://dryuc24b85zbr.cloudfront.net/tes/resources/6445534/image?width%3D500%26height%3D500%26version%3D1412632258000&imgrefurl=https://www.tes.com/teaching-resource/number-bonds-to-10-and-20-6445534&docid=eefcli0i3ZxUzM&tbnid=8VrGuew2D_7VKM:&vet=1&w=386&h=500&safe=strict&bih=463&biw=784&q=adding%20using%20numicon&ved=0ahUKEwjl06-Bn8nRAhVMKsAKHYwPCuMQMwhKKCcwJw&iact=mrc&uact=8) | *Memorise/reason with bonds to 10/20 in several forms*  *(eg 9 + 7 = 16; 16 - 7 = 9; 7 = 16 - 9).*  *Pupils should realise the effect of adding or subtracting zero - establishes +/- as related operations.*  *Pupils combine and increase numbers, counting forwards and backwards.* |
| Y2 | **TO + O**  **TO + tens**  **TO + TO**  **O + O + O**  [Show addition of two numbers can be done in any order.] | Recognise/use inverse relationship between +/- and use to check calcs and missing number problems.  Pupils use concrete objects, pictorial representations and **mental strategies**.  (eg place value boards and Base 10, Numicon) | Visual images and Practical:  58 + 30 = 88  Base ten:  12 + 15= 27  Numicon:  http://4.bp.blogspot.com/-ChAHTeJx8NE/VXGoiglxjII/AAAAAAAAHTo/N82TQZsiWU0/s1600/place%2Bvalue.JPG | | | | | Visual (efficient jumps)  35 + 47= 82    Also using Numicon and Numicon number lines.  [Also jumps can be in 10s and then1s] | | | | | | Informal mental jottings:  No number line  35 + 47 = 82  47 + 30 = 77  77 + 3 = 80  80 + 2 = 82  Partitioning  35 + 47 = 82  40 + 30 = 70  7 + 5 = 12 | | | | Adding fractions:  Use visual images and Fraction Pies/ walls. Also use Numicon. | | | | expanded written method (vertical)  *Recording addition in columns supports place value and prepares for formal written methods with larger numbers.*  47 + 35 = 82 | | Recall and use addition facts to 20 fluently.  Derive and use related facts up to 100.  Solve problems by applying increasing knowledge of mental methods. | *Pupils extend understanding of the language of + to include* ***sum***.  *Practise + to 20 to derive facts such as using 3 + 7 = 10 to calculate 30 + 70 = 100, 100 - 70 = 30 and 70 = 100 - 30. Check calculations, including by adding numbers in a different order to check addition.*  *Establishes commutativity and associativity of addition.* |
| Y3 | Use **formal** written methods of **columnar addition**.  **TO + TO**  **HTO + TO**  **HTO + HTO**  Add fractions with the same denominator. | Draw/use Number lines:  57 + 285 = 342  397+35= 432 | | No number line (jottings)  57 + 285 = 342  285 + 50 = 335    335 + 7 = 342 | | | | | **Expanded**  Vertical (used for selected children, if needed)  300 + 70 + 4  +200 + 40 + 8  600 + 110 + 12= 622  All children use base 10 and place value charts/ Numicon to introduce/support. | | | | | | | **Column**  All children use base 10 and place value charts/ Numicon to Introduce/support.  Estimate answers and use inverse to check. | | | | Add fractions: | | | | HTO + O; HTO + tens  HTO + hundreds  Use number facts and place value to solve problems.  *For mental calculations with TO nos, answers could be >100.* | *If children are taught to add TO and HTO by using columnar formal method FIRST, they may not be able to add mentally two digit numbers which is a year 2 and beyond requirement. Formal methods are most efficient for adding and subtracting larger numbers and decimal numbers.* |
| Y4 | Use **formal** written methods of **columnar addition**.  **HTO + HTO**  **ThHTO + HTO**  **ThHTO + ThHTO**  **Solve problems involving fractions** | Estimate and use inverse operations to check answers to a calculation.  Estimate, compare and calculate different measures, including money in pounds and pence. | **Expanded vertical**  (only for lower attainers/AEN)  789 + 642 = 1431  700 + 80 + 9  +600 + 40 + 2  1300 + 120 + 12= 1431  All children- concrete- use base 10 and place value charts. | | | | | | | **Column**  789 + 642 = 1431 | | ADD FRACTIONS | | | | | | | | | **Solve addition two-step problems in contexts, deciding which operations and methods to use & why.**  Solve simple measure and money problems involving fractions and decimals to 2dp | | | *Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency.* | *Pupils build on their understanding of place value and decimal notation to record metric measures, including money.* |
| **Y5** | Add whole numbers >4 digits, including using **formal** written methods (**columnar addition**).  Decimals up to 2dp  (eg 72.5 + 45.7) | **Solve problems involving number up to 3dp.**  **Solve problems involving converting between units of time. [*Measurement*]**  **Use all four operations to solve problems involving measure [eg length, mass, volume, money] using decimal notation including scaling. [*Measurement*]** | | | | Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.  Solve addition multi-step problems in contexts, deciding which operations and methods to use and why.  Add fractions with different denominators (see year 6). | | | | | | | **Column**  vertical  124.9  + 117.25  242.15  11  23.70  + 48.56  72.26    1 1 | | | | | *Pupils practise adding decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1.*  **Partitioning and recombining:**  Either partition both numbers and recombine or partition the second number only e.g.  35.8 + 7.3 = 35.8 + 7 + 0.3  = 42.8 + 0.3  = 43.1 | | | | | | Add numbers mentally with increasingly large numbers  (eg 12462 + 2300 = 14762).  *Pupils mentally add tenths, and one-digit whole numbers and tenths.* | *They extend their knowledge of fractions to thousandths and connect to decimals and measures.*  *Pupils should go beyond the measurement and money models of decimals (eg by solving puzzles.* |
| Y6 | Solve multi-step problems in contexts, deciding which operations/methods to use and why.  Decimals up to 3dp (Context: Measures) | Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. | Use knowledge of the order of operations to carry out calculations involving subtraction. | | | | **Solve problems which require answers to be rounded to specified degrees of accuracy. [*Fractions*]**  **Solve problems involving the calculation and conversion of units of measure, using decimal notation to 3dp where appropriate. [*Measurement*]**  **Column** vertical  3.243  + 18.070    21.313    1 1 | | | | | | | | **Add fractions with different denominators:** | | | | | | | | | Perform mental calculations, including with mixed operations and large numbers.  *Using the number line, pupils add positive and negative integers for measures such as temperature.* | *Pupils develop skills of rounding/estimating to predict/check order of magnitude of ans to decimal calcs. Includes rounding answers to a degree of accuracy & checking reasonableness.* |

**PART 2: SUBTRACTION R-Yr6**

#### STATUTORY EXPECTATIONS NC 2014 Rapid Recall/Mental Calculations Non-statutory guidance

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| YR | Count … from 1-20 … and say which no. is 1 less than a given no. Using quantities objects, subtract two U nos and count back to find the answer. [Expected] Estimate no. of objects; check quantities by counting up to 20.[Exceeding] | Practical (sometimes recorded using ICT or by adults).  Chloe was playing in the maths area. “I need three more” she said as she added some cubes to the circle. She then realised she had more than her friend. “Oh, I have too many”. She removed one. “Now we have the same”.  During a game of skittles outdoors Joseph knocked three numbered skittles down. He was able to calculate his score in his head.  [[EYFS Profile exemplifications, STA](http://media.education.gov.uk/assets/files/pdf/e/elg11%20numbers.pdf)] | | | | | | | | | Pictures/Objects  I have five cakes. I eat two of them. How many do I have left?    Might be recorded as: *5 – 2 = 3* | | | | | | | | | | | Symbolic Mum baked 9 biscuits. I ate 5. How many were left?  [Might be recorded as: *9 – 5 = 4]* | | | | | | | Begin to learn subtraction facts for each number to 5, extend to 10 if ready. | Children use Numicon/concrete objects: |
| Y1 | Subtract (and add) one-digit and two-digit numbers to 20 (9 + 9, 18 - 9), including zero  Read/write/interpret statements involving addition (+), subtraction (-) and equals (=) signs | Pupils use concrete objects and pictorial representations  (eg: unifix cubes, counters, Numicon and Numicon number lines, base 10, unifix)  Introduce **Bar Method**:  https://content.ncetm.org.uk/images/microsites/primary_magazine/issue_56/56_share_11.gif | **Taking away** – jumps of 1  (modelled using bead strings and numberlines)  13 – 5 =8  Also start to count on to subtract from smallest to largest number:  https://1determination2014.files.wordpress.com/2014/03/subtract-by-counting-on.png | | | | | | | | | **Taking away**  Progress to more efficient jumps 13 – 5 = 8    No number line:  13 – 3 = 10  10 – 2 = 8. | | | | | | | **Counting on** – jumps of 1  (modelled using bead strings/numberlines)  11 – 8 = 3 | | | | | | | | **Counting on** (efficient jumps)  Bridge 10:  14-8=  8+2+4=6  With, or without, number line  10-2=  8 + 2 = 10  11-1=  10 + 1 = 11 | | Represent/use number bonds and related subtraction facts within 20.  *Problems should include terms: put together, add, altogether, total, take away, distance between, more than and less than, so pupils develop concept of +/- and use operations flexibly.*  Missing number problems (eg 7 = ? – 9) | *Memorise/reason with bonds to 10/20 in several forms (eg 9 + 7 = 16; 16 - 7 = 9; 7 = 16 - 9). Pupils should realise the effect of adding or subtracting zero - establishes +/- as related operations.*  *Pupils combine and increase numbers, counting forwards and backwards.* |
| **Y2** | **TO - O**  **TO - tens**  **TO - TO**  [Show subtraction of two numbers cannot be done in any order.] | Recognise/use relationship betw. +/- to check calcs and missing number problems.  Pupils use concrete objects and pictorial representations and **mental strategies** (eg place value counters, Diene) | | | Practical/visual images  95 - 60 = 35  Use Base Ten and Numicon and numberlines. | | **Taking away** 84 - 36 = 48  [Also jumps can be in 10s/1s]  Use **Bar Method:**  http://www.teach-kids-math-by-model-method.com/images/comparison-concept-001.jpg | | | | | | | **Taking away**  (no number line)  84 - 36 = 48  84 - 30 = 54  54 - 4 = 50  50 - 2 = 48 | | | | | | **Counting on**  **83-27**  **Counting on using a blank number line:**    [Also jumps can be in 10s/1s] | | | | | *Recording subtraction in columns supports place value and prepares for formal written methods with larger numbers.*  **Expanded vertical:**  98 - 35 = 63 | | | | Recall and use subtraction facts to 20 fluently.  Derive and use related facts up to 100.  Solve problems by applying increasing knowledge of mental methods. | *Pupils extend understanding of the language of subtraction to include* ***difference****.*  *Practise subtraction to 20 to derive facts such as using 3 + 7 = 10, 10 - 7 = 3 and 7 = 10 - 3 to calculate 30 + 70 = 100, 100 - 70 = 30 and 70 = 100 - 30. Check calculations, including by adding to check subtraction.* |
| **Y3** | Use **formal** written methods of **columnar addition**  **TO - TO**  **HTO - TO**  **HTO - HTO** | **Counting on**  http://www.theschoolrun.com/sites/theschoolrun.com/files/content-images/jump_strategy_examples.png  Progress to 436 - 389 = 47 | | | | **Taking away** (no number line)  **Partitioning.:**  83-42=  80 and 3  - 40 and 2\_\_\_  40 and 1= 41  326 - 178 = 148  326 - 100 = 226  226 - 70 = 156  156 - 8 = 148 | | | | | | | 874 - 523 = 351  (no decomposition | | | | | **Expanded vertical**  Some children taught this method. Most use blank number lines and then move straight to columnar.  See guidance for example. | | | | | **Column**  932 - 457 = 475 | | | | | Estimate answers and use inverse to check.  Use **Bar Method:**  https://upload.wikimedia.org/wikipedia/commons/thumb/2/23/Comparison_model_subtraction.jpg/250px-Comparison_model_subtraction.jpg | HTU - U  HTU - tens  HTU – hundreds  Use number facts and place value to solve problems. | **Expanded Vertical:** |
| Y4 | Use **formal** written methods of **columnar subtraction**.  **HTO - HTO**  **ThHTO - TO**  **ThHTO - HTO**  **ThHTO - ThHTO** | **Counting on** 1324 - 968 = 356  Use **Bar Method:**  [Image result for Maths bar method](https://www.google.co.uk/imgres?imgurl=http://my.homecampus.com.sg/images/site/Singapore_Maths_Model_Method_Example_2.png&imgrefurl=http://my.homecampus.com.sg/What_is_Singapore_Math&docid=M9YXRGgNCspHVM&tbnid=oeHr88jLHkKdmM:&vet=1&w=343&h=120&safe=strict&bih=855&biw=1280&q=Maths%20bar%20method&ved=0ahUKEwj7u4m-sMnRAhXnL8AKHSsuDecQMwhBKB4wHg&iact=mrc&uact=8) | | | | | | Decomposition: 1374 - 968 = 406 | | | | | | | | **1374 - 968 = 406** | | | | | **Solve subtraction two-step problems in contexts, deciding which operations and methods to use and why.**  Solve simple measure and money problems involving fractions and decimals to 2dp. | | | | | Estimate and use inverse operations to check.  Estimate, compare and calculate different measures, including money in pounds and pence. | | | *Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency.* | *Pupils build on their understanding of place value and decimal notation to record metric measures, including money.* |
| **Y5** | Subtract whole numbers >4 digits, including using **formal** methods (**columnar subtraction**).  Decimals up to 2dp  (eg 72.5 - 45.7) | Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.  Solve multi-step problems in contexts, deciding which operations/methods to use and why. | | Solve problems involving number up to 3dp. [*Fractions*]  Solve problems involving converting betw. units of time.  [*Measurement*]  Solve problems involving measure [eg length, mass, volume, money] using decimal notation including scalin  [*Measurement*] | | | | | | **Counting on**  72.5 – 45.7 = 26.8  **Bar Method:**  http://www.onlinemathlearning.com/image-files/x6th-grade-math-practice_clip_image001.gif.pagespeed.ic.Q-QgU3Vmzc.png | | | | | **Taking away**  (no number line)  72.5 – 45.7  72.5 – 40 = 32.5  32.5 – 5 = 27.5  27.5 – 0.7 = 26.8 | | | | | | Column: 72.5 - 45.7 = 26.8 | | | *Pupils practise subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1.* | | | | | Subtract numbers mentally with increasingly large numbers  (eg 12462 - 2300 = 10162).  *Pupils mentally subtract tenths, and one-digit whole numbers and tenths.* | *They extend their knowledge of fractions to thousandths and connect to decimals and measures.*  *Pupils should go beyond the measurement and money models of decimals (eg by solving puzzles.* |
| **Y6** | Solve multi-step problems in contexts, deciding which operations/methods to use and why.Decimals up to 3dp (Context: Measures) | Use knowledge of the order of operations to carry out  calculations involving subtraction.  Use estimation to check answers to calculations and  determine, in the context of a problem, an appropriate  degree of accuracy. | | | | | | | **Solve problems which require answers to be rounded to specified degrees of accuracy. [*Fractions*]**  **Solve problems involving the calculation and conversion of units of measure, using decimal notation to 3dp where appropriate. [*Measurement*]** | | | | | | | | **Column** (see above)   * There was 2.5 litres in the jug. Stuart drank 385 ml. How much was left? * 18.07 km - 3.243 km * Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. | | | | | | | | | | | | Perform mental calcs, incl. with mixed operations and large numbers.  *Using the no. line, pupils subtract positive/negative integers for measures such as temperature.* | *Pupils develop skills of rounding/estimating to predict/check order of magnitude of ans to decimal calcs. Includes rounding ans to a degree of accuracy & checking reasonableness.* |

#### PART 3: MULTIPLICATION R-Yr6

#### STATUTORY EXPECTATIONS NC 2014 Rapid Recall/Mental Calculations Non-statutory guidance

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| YR | Children … solve problems, including doubling, halving and sharing. [Expected]  Solve practical problems that involve combining groups of 2/5/10. [Exceeding] | Practical/ recorded using ICT (eg digital photos / pictures on IWB)  How many 10p coins are here? How much money is that?  This domino is a double 4. How many spots does it have? | | | | | Pictures/Objects  How many socks in three pairs? | | | | | | | | | Symbolic  3 pairs, 2 socks in each pair: | | | | | | | | [http://tse1.mm.bing.net/th?&id=OIP.Md43bcd48fafe42d203098f696b7c0fcco0&w=216&h=259&c=0&pid=1.9&rs=0&p=0&r=0](http://www.bing.com/images/search?q=multiplication+using+real+objects&view=detailv2&&id=90B821329FB66A8BDB4C5BA4C033F78629488904&selectedIndex=3&ccid=1DvNSPr%2b&simid=608046376896171745&thid=OIP.Md43bcd48fafe42d203098f696b7c0fcco0) | *Use real objects and contexts to develop understanding.*  [http://tse1.mm.bing.net/th?&id=OIP.M28e68033969169c56d6f357aa4ec78c7H0&w=300&h=278&c=0&pid=1.9&rs=0&p=0&r=0](http://www.bing.com/images/search?q=multiplication+using+real+objects&view=detailv2&&id=57127545A5735ABF82A056B4805D41666BDA31B5&selectedIndex=9&ccid=KOaAM5aR&simid=608034389639497240&thid=OIP.M28e68033969169c56d6f357aa4ec78c7H0) |
| Y1 | Solve one-step problems using concrete objects, pictorial representations and arrays *(with the support of the teacher)* | Practical/recorded  Pictures/Symbolic  There are five cakes in each bag.  How many cakes are there in three bags?        Grouping counters and drawing pictures | | | | | | | | Visual (Number lines and Numicon on Number Lines)    **5** x 3 **or** 3 x **5** [two, three times] or [three groups of two]  4  5  0  15  10 | | | | | | | | | | | | Arrays  5 x 2 or 2 x 5 | | *Doubling numbers/quantities*  *Count on/back in 2s, 5s and 10s* | *Use Numicon to aid:* |
| Y2 | Calculate statements for multiplication within the multiplication tables and write them using the multiplication and equals signs.  [Show multiplication of two numbers can be done in any order.] | Pictures/Symbolic  There are four apples in each box.  How many apples in six boxes | | | | Pupils use a variety of language to describe multiplication.  Concrete- pupils use Numicon and Numicon Number Lines. | | | | Repeated addition  5 x 3 or 3 x 5, 5+5+5 or 3+3+3+3+3+3  0 3 6 9 12 15  0 5 10 15 | | | | | | | | | Arrays  6 x 4 or 4 x 6 | | | | | Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, (including recognising odd and even numbers).  *Use commutativity/inverse relations to develop multiplicative reasoning (eg 4 × 5 = 20 and 20 ÷ 5 = 4).* | *Pupils … practise to become fluent in the 2/5/10 multiplication tables and connect them to each other.*  *They connect the 10x table to place value, and the 5x table to divisions on the clock face.*  *They begin to use other multiplication tables and recall facts, including using related division facts to perform written and mental calculations.* |
| Y3 | Write/calculate statements using the multiplication tables that they know (progressing to **formal** written methods).  **TU x U**  (multiplier is 2/3/4/5/8/10) | **Grid Method:**  36 x 4 = 144 | | | **Informal Mental Jottings:**  36 x 4 = 144 | | | | | | **Expanded multiplication**:  36 x 4 = 144    (6 x 4)  (30 x 4) | | | **Short multiplication** (formal):  36 x 4 = 144 | | | | | | *Pupils develop reliable written methods for multiplication, starting with calculations of TU by U (progressing to formal written methods of short multiplication).* | | | | Recall and use multiplication facts for the 3, 4 and 8 multiplication tables. | *Through doubling, they connect the 2/4/8 multiplication tables.*  *Pupils develop efficient mental methods, using commutativity (eg 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts (eg using 3 × 2 = 6, 6 ÷ 3 = 2 & 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, 60 ÷ 3 = 20 & 20 = 60 ÷ 3).* |
| Y4 | Use **formal** written layout:  **TU x U**  **HTU x U**  Convert between different units of measure [eg km to m; hr to mi] | 43 x 6 = 258  (estimate: 40 x 6 = 240)  40 x 6 = 240  3 x 6 = 18 | **Expanded multiplication:**  43 x 6 | | | | | **Short multiplication (formal):**  24 x 6 = 144 | | | | **Grid Method:**  342 x 7 = 2394 | | | | | **Expanded multiplication**: | | | | | | **Short multiplication** (formal)  342 x 7 = 2394  Progress to **Long Multiplication** TO x TO when ready | Recall multiplication facts to 12 × 12.  Use place value, known & derived facts to multiply mentally, including x by 0/1; x 3 numbers.  Recognise/use factor pairs and commutativity in mental calculations.  *Pupils use multiplication to convert from larger to smaller units.* | *Practise mental methods and extend this to HTU numbers to derive facts, for example 200 × 3 = 600 into 600 ÷ 3 = 200.*  *Write statements about equality of expressions [eg 39 × 7 = 30 × 7 + 9 × 7 and (2 × 3) × 4 = 2 × (3 × 4)]. Combine knowledge of facts and arithmetic rules to solve mental/written calculations (eg 2 x 6 x 5 = 10 x 6 = 60).* |
| **Y5** | Use a **formal** written method (including long x for TU nos)  **TU x TU**  **HTU x U / HTU x TU**  **ThHTU x U**  Convert between units of measure (eg km/m; m/cm; cm/mm; kg/g; litre and ml)  Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams | **Grid Method:**  47 x 36 = 1692  (estimate 50 x 40 = 2000) | | **Multiply fractions:** | | | | | **Short multiplication** (formal)  2741 x 6 = 16446  (estimate 3000 x 6 = 18000 | | | | 24 x 16 = 384 (estimate 25 x 15 = 375) | | **Long Multiplication**  124 x 26 = 3224 [see Y6] | | | | | | *Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division. This relates to scaling by simple fractions, including those > 1.*  *Find fractions of numbers and quantities, writing remainders as a fraction.* | | | Identify multiples/factors, including finding all factor pairs of a number, & common factors of two numbers**.**  Know/use vocabulary of prime numbers, prime factors and composite (non-prime) nos.  Establish if a number up to 100 is prime; recall prime numbers to 19.  x nos mentally using known facts.  Multiply whole numbers and those involving decimals by 10/100/1000. | Pupils … *apply all the x tables frequently, commit them to memory and use them to make larger calculations.*  *They understand the terms factor, multiple/prime, square/cube numbers & use to construct equiv. statements (eg 4 x 35 = 2 x 2 x 35; 3 x 270 = 3 x 3 x 9 x 10 = 9² x 10).* |
| Y6 | Multi-digit numbers (up to 4 digits) x TU whole number using the **formal** method of **long multiplication**.  Multiply one-digit numbers with up to two decimal places by whole numbers | 256 x 18 = 4608  (estimate 250 x 20 = 5000) | | **Long Multiplication (formal):**  124 x 26 = 3224  [NB See Y5 method] | | | | | | Short multiplication (formal)  4.7 x 8 = 37.6  (estimate 5 x 8 = 40) [Or 47 x 8, then divide the solution by 10.] | | | Grid Method:  5.65 x 9 = 50.85  (estimate 6 x 9 = 54)  [Or compute 565 x 9, then divide the solution by 100.] | | | | | *Use a variety of images to support understanding of x with fractions. Use understanding of relationship between unit fractions and ÷ to work backwards by x a quantity that represents a unit fraction to find the whole quantity (eg if ¼ of a length is 36cm,whole length 36 × 4 = 144cm).*  *x numbers with up to 2dp by U/TU whole nos (starting with simplest cases eg 0.4 × 2 = 0.8, and in practical contexts).* | | | | | | Perform mental calculations, including with mixed operations/large numbers.  Identify common factors/multiples and prime numbers.  Use knowledge of order of operations to carry out calculations.  Use estimation to check answers to calculations and determine an appropriate degree of accuracy.  Identify value of each digit to 3dp and x nos by 10/100/1000 (ans to 3dp) | *Undertake mental calcs with increasingly large numbers and more complex calculations.*  *Continue to use all x tables to calculate statements in order to maintain their fluency.*  *Explore the order of operations using brackets.*  *Common factors can be related to finding equivalent fractions.* |

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|  | STATUTORY EXPECTATIONS NC 2014 | **PART 4: DIVISION R-Yr6**  Rapid Recall/Mental Calculations *Non-statutory guidance* | | | | | | | | | | | | | | | | | | | | |
| YR | Children … solve problems, including doubling, halving and sharing. [Expected]  They solve practical problems that involve sharing into equal groups. [Exceeding] | Practical / recorded using ICT (eg digital photos/pictures on IWB) | | Pictures/ real objects  6 cakes shared between 2  j0079104j0079103  6 cakes put into groups of 2 | | | | | | | | Symbolic  6 cakes shared between 2  [Image result for dividing by sharing](https://www.google.co.uk/imgres?imgurl=http://www.enchantedlearning.com/math/divide/visual/fishintanks.png&imgrefurl=http://www.enchantedlearning.com/math/divide/&docid=PfSnWC8ummgu6M&tbnid=v8xOyJWXd40SHM:&vet=1&w=653&h=183&safe=strict&bih=855&biw=1280&q=dividing%20by%20sharing&ved=0ahUKEwiKwc2o0MnRAhVqDMAKHbs1Aks4ZBAzCAooCDAI&iact=mrc&uact=8) | | | | | | | There are 8 raisins.  Take half of them.  How many do you have?  Share the 10 grapes between 2 people. | | https://lh6.ggpht.com/th6hn8aDIL8c0VGommaU3Act2H6VA2H4Tmhvs2LIJzDZy0Ne5crqTU9kEE45FADTmbbr=w300  https://melodyhousemusic.com/wp-content/uploads/LER-0739-Baby-Bear-Sorting-Set-300x300.jpg | *Ensure the children have lots of practical experience of sharing/ division (not worksheets)* |
| Y1 | Solve one-step problems using concrete objects, pictorial representations and arrays *(with the support of the teacher)* | There are 14 people on the bus. Half of them get off.  How many remain on the bus?  There are 20 people in the class. One quarter are boys. How many boys are there? | | | Pictures/Symbolic  How many apples in each bowl if I share 12 apples between 3 bowls? | | | | | | | Visual and Concrete (modelled using Numicon and Number lines/ beads/ counters/objects)  15 ÷ 5 = 3  0 5 10 15  https://i.ytimg.com/vi/UejjvxBKh7I/maxresdefault.jpg | | | | | | | | | Recognise/find/name ½ as one of two equal parts of an object, shape or quantity.  Recognise/find/name ¼ as one of four equal parts of an object, shape or quantity. | *Find simple fractions of objects, numbers and quantities*  *Count on/back in 2s, 5s and 10s* |
| Y2 | Calculate statements within the multiplication tables and write them using the division and equals signs. [Show division of two numbers cannot be done in any order.]  Find ⅓, ¼, ²⁄₄, ¾ of a length/objects/quantity.  Write simple fractions  eg ½ of 6 = 3 | Pictures/Symbolic  Four eggs fit in a box.  How many boxes would you need to pack 20 eggs? | | | | | Pupils use a variety of language to describe division. | | | | Visual / Concrete  (modelled using Numicon and Numberline )  Skip counting:  19 ÷ 2 = 4  http://www.mathematicsdictionary.com/english/vmd/images/s/skipcount.gif | | | | | Arrays  Find ¼ of 24  24 ÷ 4 = 6 | | | | Partitioning  32 ÷ 2 = 16  20 ÷ 2 = 10  12 ÷ 2 = 6 | Recall & use division facts for the 2, 5 and 10 multiplication tables,  Recognise/find/name/write fractions ⅓, ¼, ²⁄₄, ¾ of a (length, shape), set of objects or quantity.  Write simple fractions eg ½ of 6 = 3 and recognise equivalence of two quarters and one half.  *Use commutativity/inverse relations to develop multiplicative reasoning (eg 4 × 5 = 20 and 20 ÷ 5 = 4).* | *Begin to use other multiplication tables/recall facts, including related division facts to perform written/mental calculations.*  *Work with materials/contexts where division relate to grouping/sharing quantities. They begin to relate these to fractions/measures (eg 40 ÷ 2 = 20, 20 is a half of 40).*  *They connect unit fractions to equal sharing and grouping, to numbers and to measures* |
| Y3 | Write/calculate statements using the tables that they know (progressing to **formal** written methods).  **TU ÷ U**  (divisor is 2/3/4/5/8/10) | Revise skip counting: 24÷4 or 24÷6=  [Related image](http://www.google.co.uk/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=0ahUKEwjo1MSNusnRAhVJ7RQKHSn9CYgQjRwIBw&url=http://www.montereyinstitute.org/courses/DevelopmentalMath/COURSE_TEXT_RESOURCE/U01_L3_T1_text_final.html&bvm=bv.144224172,d.ZGg&psig=AFQjCNEItQK9gPvwgF52TgzFgNKRzENxkA&ust=1484751881667134)  Progress to chunking on a numberline  (start at 0)  96 ÷ 4 = 24  20 x 4 4 x 4    0 80 96 | | | | Multiples of the divisor  85 ÷ 5 = 17  10 x 5 = 50  7 x 5 = 35 | | | | Written (Chunking)  51 ÷ 3 =17 | | | | | Short Division (formal)  51 ÷ 3 = 17 | | | | *Pupils develop reliable written methods for division, starting with calculations of TU by U numbers (progressing to formal written methods of short division).* | | Recall and use division facts for the 3, 4 and 8 multiplication tables. | *Pupils develop efficient mental methods, using commutativity (eg 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts (eg using 3 × 2 = 6, 6 ÷ 3 = 2 & 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, 60 ÷ 3 = 20 & 20 = 60 ÷ 3).* |
| Y4 | *Pupils practise to become fluent in the* ***formal*** *written method of* ***short division*** *with exact answers [NS]*  **TU ÷ U; HTU ÷ U** | Chunking on a number line.  Multiples of the divisor  98 ÷ 7 = 14  10 x 7 = 70  4 x 7 = 28 | Short Division (formal)    98 ÷ 7 = 14 | | | | | Multiples of the divisor  252 ÷ 7 = 36  30 x 7 = 210  6 x 7 = 42 | | | | Written (Chunking)  252 ÷ 7 = 36 | | | | | Short Division (formal)  252 ÷ 7 = 36 | | | | Recall division facts to 12 × 12.  Use place value, known/derived facts to ÷ mentally, including ÷ by 1.  Find effect of dividing U/TU by 10/100, identifying the value of the digits in the answer as units/tenths/hundredths. | *Practise mental methods and extend this to HTU numbers to derive facts, for example 200 × 3 = 600 into 600 ÷ 3 = 200.*  *Relates decimal notation to division of whole number by 10 and later 100.* |
| **Y5** | Use the **formal** written method of **short division** (interpret remainders appropriately for the context).  **HTU ÷ U**  **ThHTU ÷ U**  Convert between units of measure (eg km/m; m/cm; cm/mm; kg/g; litre and ml) | Written (Chunking)  346 ÷ 8 = 43 r2 (estimate >40, <50)    Use **Bar Method** to solve division, fraction and ratio problems:  http://www.greatmathsteachingideas.com/wp-content/uploads/2014/12/Slide28-1024x768.png | | | 291 ÷ 3 = 97  (estimate: 270 ÷ 3 = 90) | | | | Short Division (formal)  432 ÷ 5 = 86 r2  (estimate: 400 ÷ 5 = 80) | | | | | Short Division (formal)  8520 ÷ 6 = 1420 | | | | *Pupils connect x by a fraction to using fractions as operators (fractions of), and to ÷. This relates to scaling by simple fractions, incl. those > 1.*  *Find fractions of numbers and quantities, writing remainders as a fraction.* | | | Identify multiples/factors, including finding all factor pairs of a number, & common factors of two numbers**.**  Know/use vocabulary of prime numbers, prime factors and composite (non-prime) nos.  Establish if a number up to 100 is prime; recall prime numbers to 19.  ÷ nos mentally using known facts.  Divide whole numbers and those involving decimals by 10/100/1000. | Pupils … *apply all the ÷ facts frequently, commit them to memory and use them to make larger calculations.*  *They understand the terms factor, multiple/prime, square/cube numbers & use to construct equivalent statements [eg 120 ÷15 = (30 x 4) ÷ 15 = 2 x 4 = 8]* |
| Y6 | Divide numbers (up to 4 digits) by TU whole number using the **formal** method of **short/long division** (interpret as approp. for the context).  Use written division methods in cases where the ans has up to 2dp.  *[Divide numbers up to 2dp by U/TU whole numbers.]* | 43.4 ÷ 7 = 6.2  (estimate 42 ÷ 7 = 6)  6 x 7 = 42  0.2 x 7 = 1.4  25.6 ÷ 7 = 3.2  (estimate >3, <4)  Use **Bar Method** to solve division and ratio problems: | | | Short Division (formal)  43.68 ÷ 7 = 6.24  (estimate: 42 ÷ 7 = 6)  [Or compute 4368 ÷ 7, then divide the solution by 100.] | | | | *Short Division (by TU)*  *496 ÷ 11 (estimate 500 ÷ 10 = 50)* | | | | Long division (NC 2014) 432÷15=    Long division involving decimals (NC 2014)  432÷ 15 = 28.8 | | | | | | | | Perform mental calculations, including with mixed operations/large numbers.  Identify common factors/multiples and prime numbers.  Use knowledge of order of operations to carry out calculations.  Use estimation to check answers to calculations and determine an appropriate degree of accuracy.  Identify value of each digit to 3dp and ÷ nos by 10/100/1000 (ans to 3dp) | *Undertake mental calcs with increasingly large numbers and more complex calculations.*  *Continue to use all table facts to calculate statements in order to maintain their fluency.*  *Explore the order of operations using brackets.*  *Common factors can be related to finding equivalent fractions.* |

**Progression in use of the Bar Method**

**EYFS to Yr 1 as Teacher feels appropriate.**

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| **Introduce Part-Part Whole, use tens frames and pictorial representations of patterns.** | | | |
|  |  | C:\Users\nathan.crook\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.Outlook\3IR2FLXR\photo 2.JPG**Use real objects, and multilink to make bar models to represent numbers.** |  |

**Year 1 and 2 (by end of Key Stage these concepts should be secure.)**

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| **Introduce idea of equivalence, missing number and multiple layers of bar model as well as applying the bar model to fractions, multiplication and division and to support understanding of inverse operations and remainders.** |
| What fractions can you see?  What fraction of the orange is each yellow piece?  If the value of the orange rod is ten, what is the value of each yellow rod?  Extension: What if the value of the orange rod is 6? 100? Etc.    I think of a number. I subtract 5. The answer is 4. What is my number?    See the source image  33 divided by 7 equals 4 remainder ?  [Image result for bonds to ten](https://www.google.co.uk/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=2ahUKEwiGoMSnnNHaAhUGbRQKHZ0jCbAQjRx6BAgAEAU&url=https://www.theschoolrun.com/what-are-number-bonds&psig=AOvVaw2FVTGRlnRCisUa6EtxUYuz&ust=1524601223392718)  Use the bar method to show bonds to ten, twenty, 100 in multiple layers to show equivalence and patterns. |

**Lower Key Stage Two (Years 3 and 4)**

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| **Continue to use Bar Method as so far introduced but also for more complex problems, two step problems and to replace informal jottings.** |
| Aiden has seven marbles and Harvey has fifteen. They decide to share them equally between them. How many do they get each?    *C:\Users\nathan.crook\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.Outlook\3IR2FLXR\photo (5).JPG*  This is an example of integer scaling?  solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.  e.g. 8 children each download 59 songs to play on their iPod. How many songs do they have altogether?  C:\Users\nathan.crook\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.Outlook\3IR2FLXR\photo (6).JPG  A computer game is £24 in the sale. This is one quarter of its original price. How much did it cost before the sale?    Remember to give a range of examples of number sentences changing the position of the = sign.  353+354+?=1000  ? +493+754 = 2000  **Start to use to show algebraic formulae**    **Upper Key Stage Two (Years 5 and 6)**  **By the end of year 5 the five main examples below will have been introduced in a range of examples for picturing multi step problems, making comparisons, demonstrating equal parts of a whole and then comparing them. Bar methods will be evident in books as a well understood and easily used model.**    **Further examples inclue… more complex algebra, accompanying written problems.**    **Children can adapt the model to draw more abstract representations**      Draw problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.  How many jugs with a capacity of 250ml could you fill with 10 litres of water?  **Ration and Proportion, percentages**    There is 20% off in a sale. The reduced price of the jeans is £36. What was the original price?  At a dance there are 4 girls to every 3 boys. There are 63 children altogether? How many girls are there?      **By year Six children should be able to sue the Bar Model to explain their working, share ideas, prove conjectures and adapt their model to draw a range of problems with fluency.** |